

PRIFYSGOL CYMRU : UNIVERSITY OF WALES

**DEGREE EXAMINATIONS 2010
ARHOLIADAU GRADD 2010**

**Swansea Metropolitan University
Prifysgol Fetropolitan Abertawe**

Faculty of Applied Design and Engineering

NAME OF COURSE/PROGRAMME: *BEng Motorcycle Engineering*

YEAR or LEVEL (state which): *Year II*

NAME OF MODULE/UNIT: *Motorcycle Dynamics*

NAME OF EXAM PAPER:
(if different to module name)

DATE OF EXAMINATION:

EXAMINATION TIMES:

Start Time:

Finish Time:

Time Allowed:

INSTRUCTIONS TO CANDIDATES:

For all questions, take $\rho_{air} = 1.167 \text{ kg m}^{-3}$

An equation sheet is provided with this exam paper

Answer 3 from 4 Questions

FACULTY REF:

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Question 1 – (Rectilinear Dynamics)

Consider a motorcycle with the following properties:

Mass with rider, $m = 195\text{kg}$

Reduced Mass, $m^* = 212\text{kg}$

Frontal Area, $A = 0.75\text{ m}^2$

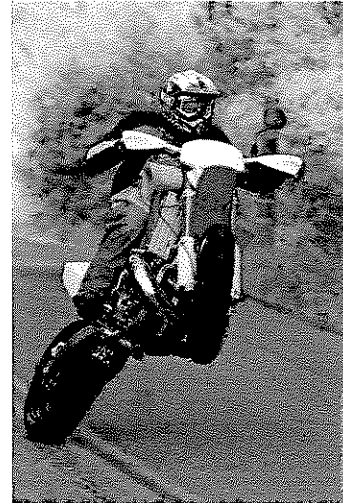
Drag Coefficient, $C_D = 0.65$

Wheelbase, $p = 1.3\text{m}$

50/50 weight distribution

Front vertical stiffness $k_f = 22\text{ N/mm}$

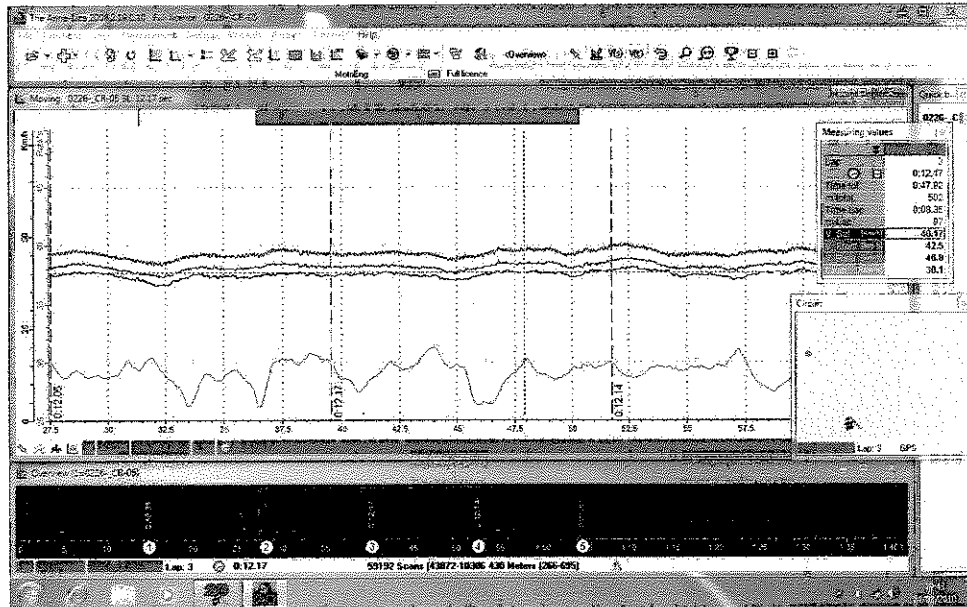
Rake angle = 22°



- What is the *total* driving force required to maintain the motorcycle at a constant velocity of 155 kmh^{-1} along a horizontal flat road?
(5 marks)
- If the same motorcycle is required to climb an incline of 30° , what power would the motorcycle be required to produce?
(3 marks)
- If the motorcycle has a maximum power output of 55KW , what would you expect its aerodynamically limited top speed to be?
(3 marks)
- If this motorcycle is capable of accelerating at $1.1g$, at what height must its centre of gravity be?
(3 marks)
- For this motorcycle in braking, its data logger shows a deceleration of $0.8g$, with a suspension compression of 80mm . Using the data provided above, show using calculations that this is a reasonable suspension compression value.
(6 marks)

Question 2 - (Steady State Cornering)

- With the aid of diagrams, derive an equation for the 'ideal' lean angle of a motorcycle in steady state cornering.
(5 marks)
- With reference to the data trace shown below:



Explain why the 3 recorded velocity traces show different values and explain which of these velocities would be suited to being used as the input to the motorcycle's traction control system?
(5 marks)

- Derive an equation, which expands on the concept of the ideal lean angle to include the motorcycle's average tyre thickness into its required lean angle to give a more accurate representation of the required lean angle.
(5 marks)
- For the motorcycle shown in the above data trace, calculate the ideal lean angle and the 'real' lean angle assuming the test track diameter to be 45m and realistic values for the required motorcycle geometry and compare your solution to the data on the trace and discuss your results.
(5 marks)

Question 3 – (Suspension Analysis)

- a. For a normal front suspension fork show that the ‘wheel rate’ and the ‘spring rate’ are related by the equation.

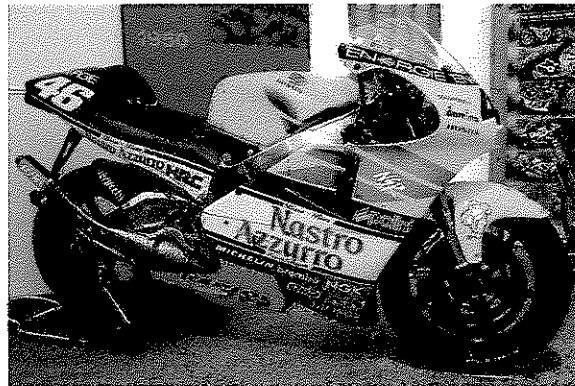
$$k = k_f \cos^2 \varepsilon$$

(5 marks)

- b. For a rear suspension system (You may assume a simplified twin-shock system), show that the ‘wheel rate’ and the ‘spring rate’ are related by the square of the link ratio.

(5 marks)

- c. The 2001 HRC NSR500 weighs 126kg wet and has a 58/42 front biased weight distribution with rider. The fork travel is 112mm, the rear wheel travel is 114mm, the rake angle is 21.5° , the front unsprung mass is 11kg and the rear unsprung mass is 14.5kg.



Advise a suitable ride frequency for the motorcycle and assuming a total weight of 200kg (Including rider) determine a suitable vertical stiffness for the motorcycle

(5 marks)

- d. Assuming that the NSR500 has a 2.2:1 linear link ratio at the rear, determine the required spring stiffness's for the front and rear of the motorcycle.

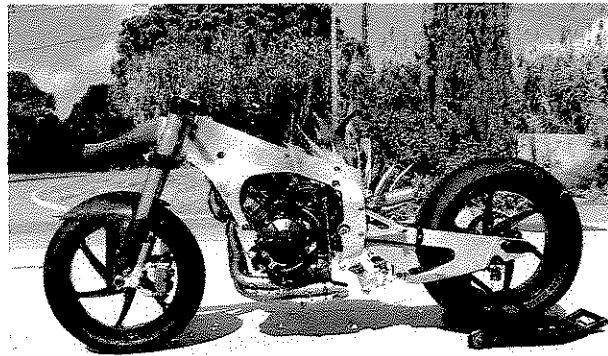
(5 marks)

Question 4 – (Dynamics and Damping)

- a. Derive a set of equations that can be used to determine the wheel loads of a motorcycle in both static and dynamic situations.

(8 marks)

The Tech3 Moto 2 motorcycle has a total mass of 144kg and a 58/42 forward weight distribution with a 70kg rider. It has a front unsprung mass of 10.4kg and a rear unsprung mass of 14.3kg. The rake angle is 22.5 degree and the rear link ratio is an average of 2.5:1. Determine the following:



- b. Determine the vertical wheel stiffness's that are needed to satisfy a ride frequency of 2.0 Hz for the front and 2.2Hz for the rear.

(3 marks)

- c. Determine the critical damping that would be needed for these vertical stiffness's?

(3 marks)

- d. Given the suspension geometry for the motorcycle, determine the damping required at the fork and the shock for the sprung mass for both the front and rear of the motorcycle if the damping ratio for the sprung mass is to be $\zeta = 0.4$.

(6 marks)

Equation Data Sheet

Dynamic Loadings:

$$N_f = mg \frac{b}{p} - S \frac{h}{p}$$

$$N_R = mg \frac{(p-b)}{p} + S \frac{h}{p}$$

Aerodynamic Drag:

$$Force = \frac{1}{2} \rho C_D A V^2$$

Maximum Acceleration

$$\ddot{x} = \frac{mg}{m^*} \frac{b}{h} - \frac{F_D}{m^*}$$

Maximum Deceleration

$$\ddot{x} = \frac{g(p-b)}{h} - \frac{F_D}{m}$$

Ideal Roll Angle

$$\varphi_i = \tan^{-1} \left(\frac{R_c \Omega^2}{g} \right)$$

Tyre Correction

$$\frac{\sin \Delta \varphi}{t} = \frac{\sin \varphi_i}{h-t}$$

Natural Frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$$



EXAMINATION SOLUTIONS : SESSION 2009/10

Programme:	BEng Motorcycle Engineering	Year:	Level 5
Module/Unit:	Motorcycle Dynamics		
Author:	Dr Owen Williams	Page:	1 of 8

Question No	Question	Marks
1a	Total driving Force is Drag + frictional + gravity (for slope) $155 \text{ km/h} = 43.05 \text{ m/s}$ so force = $\frac{1}{2} \rho C_D A V^2 + 0.02 mg$ $= (\frac{1}{2} \rho \times 0.65 \times 0.75 \times 43.05^2) + (38.26)$ $= 490 \text{ N}$	5
b	Including a 30° incline ... force = $490 + mg \sin 30$ $= 490 + 956.5$ $= 1446.5 \text{ N}$	3
c	Power = $\frac{1}{2} \rho C_D A V^3$ $V^3 = \frac{55000}{0.5 \times 1.167 \times 0.85 \times 0.75}$ $V^3 = 193351.35$ $V = 57.8 \text{ m/s}$ or 208 km/h	3

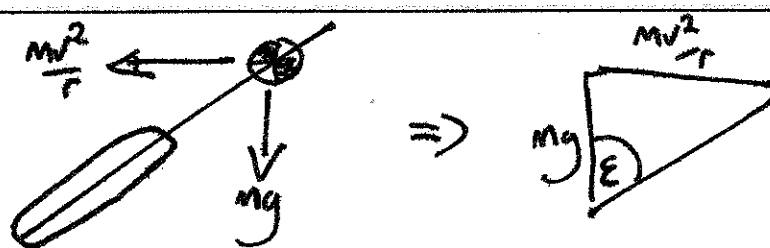
Continuation Sheet

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Question No	Question	Marks
d	if Capable of 1.1g acceleration: $\ddot{x} = \frac{Mgb}{M^*h} - \frac{F_D}{M^*}$ $1.1g + \frac{\frac{1}{2}\rho C_D A V^2}{M^*} = \frac{Mgb}{M^*h}$ $h = \frac{Mgb}{M^* \times 1.1g} = \frac{1243.4}{2287.7}$ $h = \underline{0.544 \text{ m}}$	3
e	$N_f = Mg \frac{b}{p} + S \frac{h}{p} \quad \left(S \text{ now +ve due to braking} \right)$ $= Mg \left(\frac{1}{2} \right) + 0.8g \times \left(\frac{0.544}{1.3} \right) \times m$ $= \underline{\underline{1596.8 \text{ N}}}$ If $N_f = 1596.8 \text{ N}$ $N_f = K_f x$ $x = \frac{N_f}{K_f} = \frac{1596.8}{22} = 72.58 \text{ mm}$ as forks are inclined at 22° real travel is more $\text{So } \frac{72.58}{\cos 22} = 78.28 \text{ mm} \approx 80 \text{ mm}$	6
		20

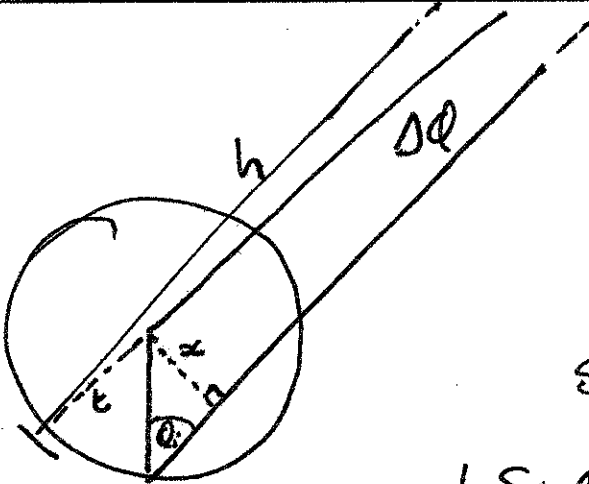
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Question No	Question	Marks
2a	 $\tan \epsilon = \frac{mv^2}{r \cdot mg}$ $\epsilon = \tan^{-1} \left(\frac{v^2}{r \cdot g} \right)$ Solution in radians is also fine. $\epsilon = \tan^{-1} \left(\frac{\Omega^2 r}{g} \right)$	5
2b	The 3 braces are different due to the thickness of the tyre and its effect on rotational speed at Roll angles V_{Rear} is best suited to a traction control input referential to V_{Front} - GPS speed can drop out due to terrain + other factors so is not suitable.	5

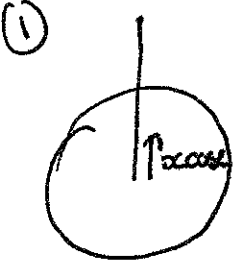
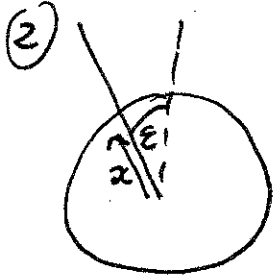
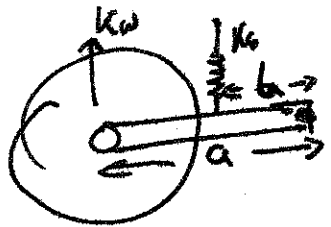
Continuation Sheet

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Question No	Question	Marks
2c	 $\sin Q = \frac{x}{t}$ $\sin \Delta Q = \frac{x}{h-t}$ Solve in $t \sin Q = (h-t) \sin \Delta Q$ $\frac{\sin Q}{h-t} = \frac{\sin \Delta Q}{t}$	5
d	$Q_i = \tan^{-1} \left(\frac{v^2}{rg} \right) \Rightarrow Q_i = 29.2^\circ$ $\sin \Delta Q = \frac{t \sin 29.2}{h-t} = 0.0629$ $\Delta Q = 3.61^\circ$ So Real lean angle = 32.8° This is similar to the real data here, but overestimates due to assumption of constant tyre thickness, front + back and other inaccuracies	5
		20

Continuation Sheet

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Question No	Question	Marks
3a	(1)  $N_f = K_f x \cos \epsilon$ (2)  $N_f \cos \epsilon = Kx$ Solve $K_f x \cos \epsilon \cos \epsilon = Kx$ <u>$K_f \cos^2 \epsilon = K$</u> b/  From $F = Kx$ $F_s = \frac{b}{a} F_w$ $x_s = \frac{a}{b} x_w$ $K_s = \frac{F_s}{x_s} = \frac{F_w \frac{b}{a}}{x_w \frac{a}{b}}$ $K_s = K_w \times \left(\frac{b}{a}\right)^2$ ← link Ratio Squared c/ Ride frequency = 2.2 Hz $K = (2\pi f)^2 \times M$ $= (2\pi \times 2.2) \times 105$ (mass $\times 0.58$ - unsprung) <u>$= 20 \text{ N/mm}$</u>	5
		5

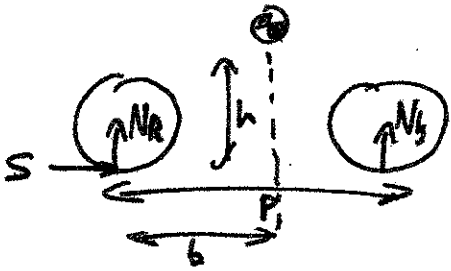
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Question No	Question	Marks
3d	Front Vertical : 20 N/mm Rear Vertical : $(2\pi f) \times m = 15.5 \text{ N/mm}$ Front Stiffness $K = K_f \cos^2 \theta$ $K = 20 \times \cos^2 21.5$ $K = 17.31 \text{ N/mm}$ Fork Spring Stiffness = $\frac{17.31}{2} = 8.65 \text{ N/mm}$ <u>in Real terms 8.5 N/mm</u> Rear Stiffness $K = K_f \times L R^2$ $K = 15.5 \times 2.2^2$ <u>$K = 75 \text{ N/mm Rear Spring}$</u>	5 20

Continuation Sheet

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Question No	Question	Marks
4a	 $N_R + N_f = mg$ Take moments about C of G. $N_f(P-b) + Sh = N_R b$ $N_f(P-b) + Sh = mgb - N_f b$ $N_f P - N_f b + Sh = mgb - N_f b$ $N_f P = mgb - Sh$ <u>$N_f = mg \frac{b}{P} - S \frac{h}{P}$</u> by similar method $N_R = mg \frac{(P-b)}{P} + S \frac{h}{P}$	8
4b	Total mass = 214 kg front US = 113.72 kg Rear US = 75.58 kg front = $(2\pi f)^2 \times m = 17.96 \text{ N/mm}$ Rear = 14.44 N/mm	3

Continuation Sheet

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Question No	Question	Marks
4c	Critical Damping = $2\sqrt{km}$ Front = $2\sqrt{174960 \times 113.72} = \underline{2858 \text{ Ns/m}}$ Rear = $2\sqrt{144444 \times 75.58} = \underline{2089 \text{ Ns/m}}$	3
4d	damp at Fork $2d = d_v \cos^2 \epsilon \times 0.4$ $d_f = \frac{1}{2} \times 2858 \times \cos^2 22.5 \times 0.4$ $d_f = \underline{487.9 \text{ Ns/m}}$ one fork leg Damp at Shock $d_r = d_v \times LR^2 \times 0.4$ $d_r = 2089 \times 2.5^2 \times 0.4$ $d_r = \underline{5222.5 \text{ Ns/m}}$ for Rear Shock	6
		20